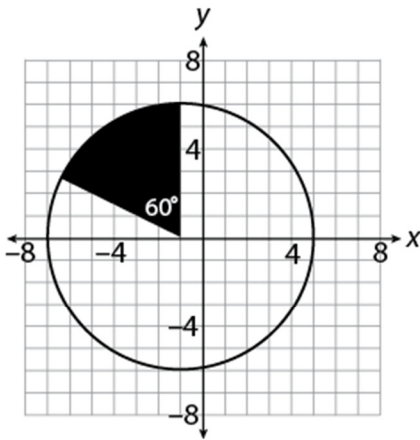


1.



What is the area of the shaded sector of the circle shown in the figure above?

- (A)  $2\pi$
- (B)  $6\pi$
- (C)  $12\pi$
- (D)  $36\pi$

**Difficulty:** Medium

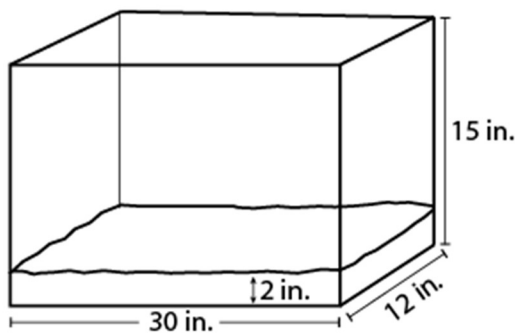
**Category:** Additional Topics in Math / Geometry

**Strategic Advice:** The area of a sector is equal to the area of the circle times the fraction of the circle represented by the sector.

**Getting to the Answer:** Start by finding the area of the whole circle: The diameter of the circle extends along the x-axis from -7 to 5, which is 12 units, which means the radius is 6. Substitute this into the area formula:

$$\begin{aligned}
 A &= \pi r^2 \\
 &= \pi(6)^2 \\
 &= 36\pi
 \end{aligned}$$

2.



The figure above shows a fish tank with sand in the bottom. If the water level is to be 3 inches below the top, how many cubic inches of water are needed to fill the tank?

**Difficulty:** Medium

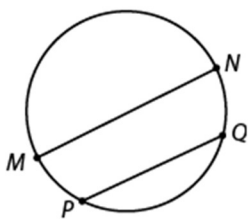
**Category:** Additional Topics in Math / Geometry

**Strategic Advice:** Determine the dimensions of the tank in which there will be water. Then, use the formula for finding the volume of a rectangular prism: Volume = length  $\times$  width  $\times$  height.

**Getting to the Answer:** The tank is 30 inches long, 15 inches tall, and 12 inches wide. The sand and the space left at the top of the tank do not affect the length or the width, only the height of the water. There are 2 inches of sand in the bottom and 3 inches of space left at the top, which means the height of the water is  $15 - 2 - 3 = 10$  inches. Use the formula Volume =  $l \times w \times h = 30 \times 12 \times 10$ .

To multiply the numbers without a calculator, multiply  $3 \times 1 \times 12 = 36$  and then add two zeros to get 3,600 cubic inches of water.

3.



The circle shown has a radius of  $r$  centimeters. If chord  $PQ$  is parallel to diameter  $MN$ , and the length of chord  $PQ$  is  $\frac{3}{4}$  of the length of the diameter, what is the distance in centimeters between chords  $MN$  and  $PQ$  in terms of  $r$ ?

(A)  $\frac{\sqrt{7}}{4}r$

(B)  $\frac{\sqrt{3}}{2}r$

(C)  $\frac{1}{4}\pi r$

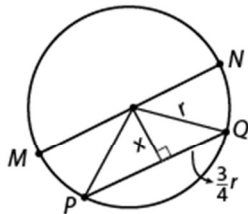
(D)  $\frac{3}{4}\pi r$

**Difficulty:** Hard

**Category:** Additional Topics in Math / Geometry

**Strategic Advice:** Drawing in a radius or two is usually a good way to start a circle question, especially when there doesn't seem to be a lot of information given. This question asks about the distance between the chord and the diameter, so start by drawing that in. Then, see if drawing a radius will help.

**Getting to the Answer:** After you've drawn in anything that you think might help you answer the question, go back and label wherever possible. The radius has length  $r$ , so add that to the diagram. The chord is  $\frac{3}{4}$  as long as the diameter, which means half the chord is  $\frac{3}{4}$  as long as the radius, so add that to the diagram. You are looking for the distance between the chord and the diameter, so call that  $x$ .



You now have a right triangle with enough labels to use the Pythagorean theorem.

$$a^2 + b^2 = c^2$$

$$x^2 + \left(\frac{3}{4}r\right)^2 = r^2$$

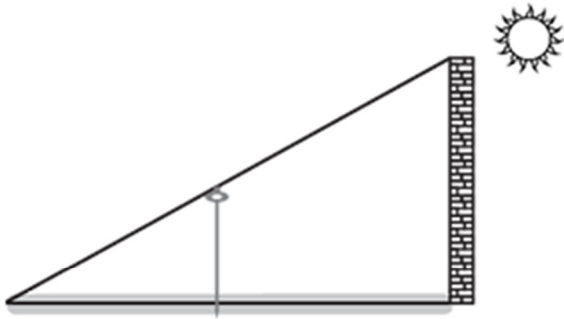
$$x^2 + \frac{9}{16}r^2 = r^2$$

$$x^2 = \frac{16}{16}r^2 - \frac{9}{16}r^2$$

$$x^2 = \frac{7}{16}r^2$$

$$x = \sqrt{\frac{7}{16}r^2} = \frac{\sqrt{7}}{4}r$$

4.



Note: Figure not drawn to scale.

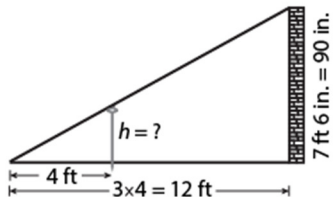
A toy saber is stuck at a right angle into the ground 4 inches deep. It casts a shadow that is 4 feet long. The brick wall casts a shadow three times that long. If the wall is 7 feet 6 inches tall, how many inches long is the toy saber?

**Difficulty:** Hard

**Category:** Additional Topics in Math / Geometry

**Strategic Advice:** Drawing on the diagram is a great strategy to get started on a question like this. There are two right triangles—the smaller one formed by the saber, the path of the sun's rays, and the ground; and the larger one formed by the brick wall, the path of the sun's rays, and the ground. The two triangles share one angle (the small angle on the left side), and each has a 90-degree angle (where the saber and the brick wall each meet the ground), making the third pair of corresponding angles also congruent. This means the triangles are similar by AAA, and the sides of the triangles are proportional.

**Getting to the Answer:** Add information from the question to the diagram. You'll need to convert the height of the wall to inches because the question asks for the length of the saber in inches. (You could also convert the base lengths to inches, but it is not necessary because you can compare feet to feet in that ratio.)

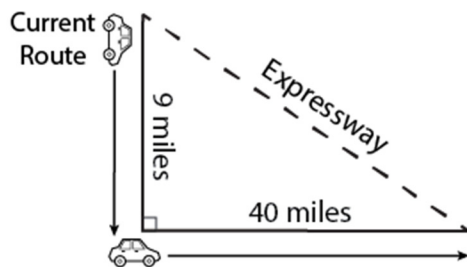


Now that you have a more detailed drawing, set up and solve a proportion:

$$\frac{\text{base of small triangle}}{\text{base of large triangle}} = \frac{\text{length of saber (in inches)}}{\text{height of wall (in inches)}}$$
$$\frac{4}{12} = \frac{h}{90}$$
$$4(90) = 12h$$
$$360 = 12h$$
$$30 = h$$

Don't forget to add the 4 inches that are stuck in the ground to find that the length of the saber is  $30 + 4 = 34$  inches.

## 5. Calculator



Note: Figure not drawn to scale.

The figure above shows the route that Max currently takes to work and back home every day. The city is planning to build an expressway that would cross through the city to help alleviate commuter traffic. Assuming an average gas consumption of 20 miles per gallon and a 5-day workweek, how many gallons of gas will Max save per week by taking the expressway to and from work each day instead of using his current route?

- (A) 2
- (B) 4
- (C) 8
- (D) 10.25

**Difficulty:** Medium

**Category:** Additional Topics / Geometry

**Strategic Advice:** It will save valuable time on Test Day if you can recognize the Pythagorean triple in this problem. If not, just use the Pythagorean theorem to find the length of the expressway.

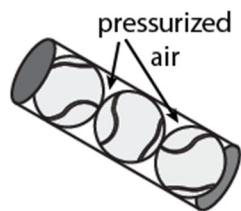
**Getting to the Answer:** The roads form a right triangle with the expressway as the hypotenuse. The two legs are Max's current route. He travels on one road for 9 miles and the other for 40. You might recognize this as a Pythagorean triple, 9, 40, 41. Even if you don't, you can always use the Pythagorean theorem to solve for the length of the hypotenuse.

$$\begin{aligned}a^2 + b^2 &= c^2 \\(9)^2 + (40)^2 &= c^2 \\81 + 1,600 &= c^2 \\1,681 &= c^2 \\41 &= c\end{aligned}$$

Now that you know the length of the expressway, it's time to analyze what the question is actually asking.

The question asks how much gas he will save given that his car gets 20 miles per gallon. His current *round-trip* route is  $2(9 + 40) = 2(49) = 98$  miles, which will use  $98 \div 20 = 4.9$  gallons of gas per day, which is equal to  $5(4.9) = 24.5$  gallons per workweek. The *round-trip* expressway route is  $2(41) = 82$  miles, which will use  $82 \div 20 = 4.1$  gallons of gas per day, which is equal to  $5(4.1) = 20.5$  gallons per workweek. Thus, he will save  $24.5 - 20.5 = 4$  gallons of gas per week by taking the expressway.

## 6. Calculator



Higher-quality tennis balls are typically packaged in cylindrical cans, as shown above, which are pressurized with air to keep them fresh. If the can and the tennis balls have the same diameter, 2.6 inches, what is the volume in cubic inches of the air inside the can around the tennis balls? Assume that each tennis ball is tangent to the next and that the top and bottom tennis balls are tangent to the top and bottom of the can.

- (A)  $4.4\pi$
- (B)  $8.1\pi$
- (C)  $10.3\pi$
- (D)  $29.3\pi$

**Difficulty:** Hard

**Category:** Additional Topics / Geometry

**Strategic Advice:** Don't forget to refer to the formula page on Test Day. The can is a cylinder and a tennis ball is a sphere, so you'll need to use both equations.

**Getting to the Answer:** Make a plan before you start plugging values into the formulas: The volume of the air is equal to the volume of the can minus the volume of the three tennis balls. For both formulas, you will need the radius. The diameters of the cylinder and the balls are the same, 2.6, but you need the radius, so divide by 2 to get  $2.6 \div 2 = 1.3$ . For the cylinder you also need the height. Because there are 3 tennis balls and the top and bottom balls are tangent to the top and bottom of the can, the height is simply the diameter multiplied by 3, which is  $2.6 \times 3 = 7.8$ . Now you're ready to use the formulas. A quick peek at the answer choices will tell you that you don't need to simplify completely.

First, find the volume of the whole can:

$$V = \pi r^2 h$$

$$V = \pi(1.3)^2(7.8)$$

$$V = \pi(1.69)(7.8)$$

$$V = 13.182\pi$$

Next, find the volume of three tennis balls:

$$V = 3\left(\frac{4}{3}\pi r^3\right)$$

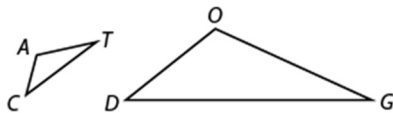
$$V = 3\left(\frac{4}{3}\pi(1.3)^3\right)$$

$$V = 4\pi(2.197)$$

$$V = 8.788\pi$$

Finally, subtract to get  $13.182\pi - 8.788\pi = 4.394\pi$ , or about  $4.4\pi$  cubic inches of air.

7.



**Note:** Figure not drawn to scale.

**Note:** Figure not drawn to scale. If triangle  $CAT$  shown above is similar to triangle  $DOG$ , and the ratio of the length of side  $TC$  to side  $GD$  is  $2:7$ , which of the following ratios must also be equal to  $2:7$ ?

- (A)  $\overline{CA} : \overline{DG}$
- (B)  $m\angle C : m\angle D$
- (C) area of  $\triangle CAT$  : area of  $\triangle DOG$
- (D) perimeter of  $\triangle CAT$  : perimeter of  $\triangle DOG$

**Difficulty:** Medium

**Category:** Additional Topics in Math / Geometry

**Strategic Advice:** Corresponding sides of similar triangles are proportional, and corresponding angles are congruent.

**Getting to the Answer:** You can eliminate B immediately because corresponding angles of similar triangles are congruent, so they are always in a  $1:1$  ratio. You can also eliminate A because side  $CA$  does not correspond to side  $DG$  ( $CA$  corresponds to  $DO$ ), so you cannot say that they will be in the same ratio. Because the side lengths are proportional, when you add the lengths of all the side lengths (the perimeter), this number will be in the same proportion, so (D) is correct. You can check this by assigning numbers that are in the ratio  $2:7$  and finding the perimeter of each triangle:

$$TC = 2 \text{ and } GD = 7$$

$$CA = 4 \text{ and } DO = 14$$

$$AT = 6 \text{ and } OG = 21$$

$$\text{Perimeter of triangle } CAT = 2 + 4 + 6 = 12$$

$$\text{Perimeter of triangle } DOG = 7 + 14 + 21 = 42$$

$$12:42 = 2:7$$